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Some system for map generation

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0. Introduction.

Recently, there have been published a lot of interesting papers on map generating systems. (For example, see papers in [1] and [2].) They also include the cell division systems which are motivated from a biological point of view. A map is defined combinatorially by Tutte [3]. Since the language theory of strings had been proposed, it has made surprising progress and now has established its proper research field. Meanwhile, the theory of two-dimensional languages is an active area as the fundamental theory of image processing. Further, graph grammars are also extremely interesting research topics with numerous applications. As seen from the definitions in the following section. map generating systems are considered as an intermediate one between the usual string languages and graph grammars. It is well-known that for the case of strings there are typical classes such as context-free, context-sensitive, and the tytpe 0. Also, interesting results of various decision problems in these systems have been shown.

In this paper, we propose some map generating systems based on string generation and solve decision problems for these systems. That is, new types of binary, propagating map generating systems are introduced. The first one is binary, propagating map 0L system with makers (mBPM0L systems). The second one is binary, propagating map IL system with makers (mBPMIL systems). After defining the two systems, we consider the following decision problems (1) and (2) on these systems:

(1) Whether or not an arbitrary mBPM0L system is deterministic ?

(2) Whether or not an arbitrary mBPMIL system is deterministic ?
These decision problems are solved. Further, decision problems of stability in these two systems are discussed. Finally, some relevant remarks related to map generating systems are given.

1. Basic definitions.

In [4], A. Lindenmayer and G. Rozenbeg introduced binary, propagating map OL systems (BPMOL systems) and discussed some properties of these systems. Further studies and applications are presented in [5]. In this paper, we proposed a new version of BPMOL systems called "BPMOL systems with makers", in which a new type of wall production rules are used. Also, we proposed BPMIL systems with makers which is similarly defined to BPMOL with makers. Here, we use the term "edge" in place of "wall" in the previous paper, and the term "wall" in place of "cell". This change of terminology became necessary in order to be consistent with the definition in [6] of three-dimensional (cellwork) L systems.

Definition 1.1 A BPMOL system with makers (mBPMOL system) G consists of

(1) a finite edge alphabet Σ,
(2) a finite set of edge productions P,
(3) a starting map ω, and
(4) the special symbols: makers $\downarrow$ and $\uparrow$, signs + and -, and opening and closing parentheses.

Here, edge productions are of the form A --> α, where A is a member of Σ(an edge label) and α is a sequence of members of Σ and signs, and of markers and matched parentheses. In this case, only one symbol appears within a closed parentheses. For instance, the rewriting of an edge represented in Figure 1 is written as the edge production:

$$A \dashrightarrow D^+C^-\downarrow(E^-)B^+.$$

The maker $\downarrow$ indicates the place and the direction (to the left or right of the original edge according its orientation) in which a new edge can be inserted. The edge symbol and sign between parentheses associated with the maker indicates the label and orientation of the new edge (+ if the orientation agree with the arrow, and - if it is opposite).

In these systems, a derivation step consists of the rewriting of all the edges surrounding a given wall, of finding the makers pointing inwards into the wall, and of inserting one new edge if there are at least two makers with matching labels and orientation present. The insertion of a new edge is uniquely determined if there are exactly two such matching makers, and the insertion is nondeterministic if there are more than three such makers. No explicit wall productions are needed in mBPMOL systems, in contrast to those without makers where edge insertion (cBPMOL systems), as in [4] and [5].

Also, no wall labels are specified in mBPMOL systems: the production of new walls depends entirely on the generation of the makers according to the edge productions. In this way, the definition of these new systems is considerably simpler than that of previous one.

Definition 1.1 is based on "zero-sided" (informationless, or interactionless) Lindenmayer systems (OL systems). This definition is naturally extended to Lindenmayer systems with interactions (IL systems). In the IL system, a $<k,\ell>$ system is generally defined, where k and ℓ are nonnegative integers. In this paper, however, we do not specialize the values of k and ℓ but consider in the general form.

<u>Definition 1.2</u> A BPMIL system with makers (mBPMIL system) is a quadruple $G=(\Sigma, \{\downarrow, \uparrow, +, -, (,)\}, P, \omega)$ which is the same as in Definition 1.1 except the following production P: edge productions are of the form $(\alpha_1, A, \alpha_2) \dashrightarrow \beta$, where $A \in \Sigma$, $\alpha_1 \subseteq \Sigma^k$, $\alpha_2 \subseteq \Sigma^\ell$, for some k, $\ell \geqq 0$, and β is a sequence of members of Σ and $\{\downarrow, \uparrow, +, -, (,)\}$.

To more easily understand production rules of the form $(\alpha_1, A, \alpha_2) \dashrightarrow \beta$ in our system, we give the following notice: Let us consider a $<1,1>$ system. In this case, $(\alpha_1, A, \alpha_2) \dashrightarrow \beta$ has generally the form represented in Figure 2.

We show here two examples for mBPMOL systems, differing only in the direction of one maker.

Case I, edge productions:

$A \dashrightarrow E^+\downarrow(G^+)F^-\uparrow(G^-)$

$B \dashrightarrow B^+C^+\downarrow(G^-)$

$C \dashrightarrow D^+.$

In Figure 3, only two makers point inward (one with G^+ and the other with G^-) and thus can serve for the placing an edge. The third maker points out wards. Here, notice that if every pair of makers is inconsistent is not produced.

Case II, edge productions:

$A \dashrightarrow E^{+}\downarrow(G^{+})F^{-}\downarrow(G^{-})$

$B \dashrightarrow B^{+}C^{+}\downarrow(G^{-})$

$C \dashrightarrow D^{+}$

In case II, wall production is nondeterministic (see Figure 4), we can insert a new edge in two positions (the third possible connection between makers is ruled out by the fact that two of the labels are G^{-} on the inward positioning makers).

The new types of wall production introduced here are motivated biologically by cell division control mechanisms based on attachment sites on edges for preprophase bands of microtubules [6].

An mBPMOL system is said to be deterministic if all derivation steps are deterministic. Of course, in a deterministic nBPMOL system there is a single position for the insertion of a new edge. When an mBPMOL system is not deterministic, it is said nondeterministic. A deterministic mBPMOL system produces a unique sequence of maps.

The language of an mBPMOL system consists of all maps generated begining with the starting map, whether deterministically or nondeterministically.

Similar terminologies and notations are introduced for mBPMIL systems.

Deterministic systems are a special case of nondeterministic ones. In general, mBPMOL (mBPMIL) systems mean nondeterministic mBPMOL (mBPMIL) systems. When we need to distinguish those two systems, deterministic and nondeterministic ones are denoted by DmBPMOL and NmBPMOL, respectively. Similar notations are used for mBPMIL systems. Also, the class of languages of mBPMOL systems is denoted by $\mathcal{L}$(mBPMOL) in usual way. For the class of languages of other systems we use the similar notations.

Now, we introduce the concept of stability for our systems G's.

Definition 1.3 Let G be a DmBPMOL (or NmBPMOL, DmBPMIL, NmBPMIL). G is said to be stable iff there does not occur any wall division in all derivation after some step.

From the above definition, if a system G is stable the the wall division by G eventually stops.

2. Decision problems

We now consider the following decision problems (1) and (2) on the systems defined in Section 1.

(1) Whether or not an arbitrary mBPMOL system is determinictic ?

(2) Whether or not an arbitrary mBPMIL system is determinictic ?

Theorem 2.1 The problem to decide whether or not an arbitrary mBPMOL system is deterministic is silvable.

Now, let us turn to the decision problem for the case of mBPMIL systems.

Theorem 2.2 The problem to decide whether or not an arbitrary nBPMIL system is deterministic is unsolvable.

Now, we consider the stability theorem.

Theorem 2.3 The problem to decide whether or not an arbitrary DmBPMOL system is stable is solvable.

Theorem 2.4 The problem to decide whether or not an arbitrary DmBPMIL system is unsolvable.

Corollary 2.5 The problem to decide whether or not an arbitrary NmBPMIL system is stable is unsolvable.

3. Remark

In this paper, we have proposed new types of map generatingsystems and showed results of some decision problems of these systems. It seems that there is a hierarchy among these systems. Also, it is an interesting topic to search for relationships between our systems mBPMOL, MBPMIL, and the cBPMOL, cBPMIL systems. We will discuss these problems in further papers.

References

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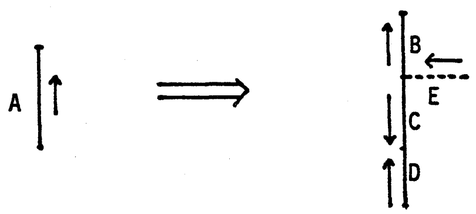

Fig.1

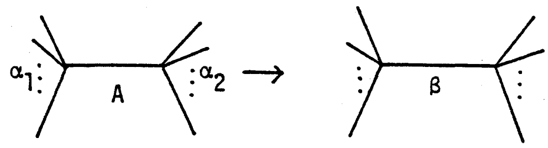

Fig.2

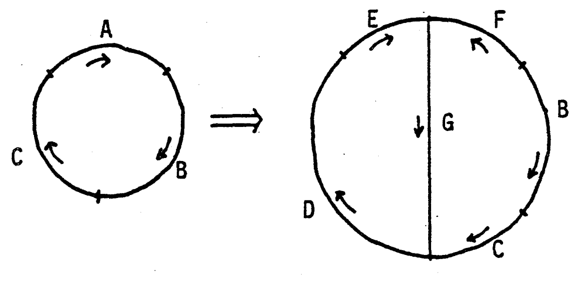

Fig.3

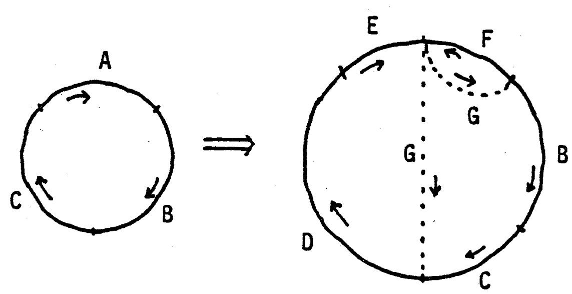

Fig.4